ECON 594: Applied Economics

Panel Data and Difference-in-Differences

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Where we are

- Yesterday: how to use AI
- Today: panel data and difference-in-differences
- Next class: event studies (the dynamic version of DD)
- These are review lectures
  - I assume you've seen this before
  - Focus is on the practical decisions you'll make in your thesis

Why panel data?

- Running example: minimum legal drinking age (MLDA) and mortality
  - Some US states had MLDA = 18 in the 1970s, others MLDA = 21
  - Y_{st}: deaths per 100,000 among 18–20 year olds in state s, year t

- Cross-sectional regression:

$$Y_{st} = \beta \cdot \mathbb{1}[\text{MLDA} < 21]_{st} + \varepsilon_{st}$$

- OLS requires $E[\varepsilon_{st} \mid \text{MLDA}_{st}] = 0$
  - Drinking culture, road quality, demographics all live in ε
  - And they are almost certainly correlated with state policy

- Panel data lets us split: $\varepsilon_{st} = \alpha_s + \nu_{st}$
  - Just control for α_s!
  - α_s absorbs anything fixed about state s
  - Now only need ν_{st} uncorrelated with policy after netting out the state

Types of data

- Cross-section: many units at one point in time
- Repeated cross-section: new random sample each period
  - E.g. American Community Survey, Canadian Census
- Panel / longitudinal: same units across many periods
  - E.g. PSID, NLSY, administrative data with stable identifiers
- Quasi-panel: repeated cross-section collapsed to group means
  - E.g. state-by-year averages from the Current Population Survey
  - Treat the group $\times$ year as the panel "unit"

Notation

- Two subscripts: unit (i or s) and time (t)
- Y_{st} = outcome for state s in year t
- Variables come in three flavours:
  - Unit-only: X_s (e.g. state area in km^2)
  - Time-only: X_t (e.g. federal funds rate)
  - Both: X_{st} (e.g. state-level policy in year t)
- Useful exercise: for every variable in your data, ask which subscripts it can have
  - Determines what variation it carries
  - Determines what fixed effects can absorb it

The fixed effects model

- Two ways to write the same regression:

With a state-specific intercept α_s:

$$Y_{st} = \beta X_{st} + \alpha_s + \nu_{st}$$

With a dummy for every state but one:

$$Y_{st} = \alpha + \beta X_{st} + \sum_{k=2}^{N} \gamma_k \mathbb{1}[s = k] + \nu_{st}$$

- These produce the same $\hat{\beta}$
- The α_s notation is just shorthand for the dummy regression
- "Fixed effect on state s" = "dummy variable for state s"

The within transformation

- Take state-level means of the regression:

$$\overline{Y}_s = \beta \overline{X}_s + \alpha_s + \overline{\nu}_s$$

- Subtract from the original:

$$Y_{st} - \overline{Y}_s = \beta(X_{st} - \overline{X}_s) + (\nu_{st} - \overline{\nu}_s)$$

- α_s has dropped out. OLS on the demeaned data gives the same $\hat{\beta}$ again
- Nice way to think about what an FE regression actually does
- This is what `reghdfe` does internally with millions of units

What you cannot identify

- Anything that doesn't vary within a state gets demeaned to zero:

$$Y_{st} - \overline{Y}_s = \beta_1 (X_{1,st} - \overline{X}_{1,s}) + \beta_2 \underbrace{(X_{2,s} - X_{2,s})}_{=0} + \cdots$$

- β_2 on a time-invariant regressor is not identified once you include state FE
- Examples: state area, distance to coast, founding year
- If you care about the effect of something fixed within state, FE can't help you
  - Often a sign you need a different research design

Stata: `reghdfe`

- `reghdfe` (high-dimensional FE) is the workhorse
- Absorbs FE without estimating them, scales to millions of units

```
ssc install reghdfe

* One-way FE
reghdfe y x, absorb(state) cluster(state)

* Two-way FE
reghdfe y x, absorb(state year) cluster(state)

* Three-way FE (firm, worker, year)
reghdfe y x, absorb(firm worker year) cluster(firm)
```

- `areg` works for one set of FE
- `xtreg, fe` works but is slower for large panels

The simplest case: 2×2 DD

- Alabama lowered its drinking age from 21 to 19 in 1975
- Neighbouring Arkansas kept its drinking age at 21
- Two groups, two periods:

	Pre (1974)	Post (1976)
Alabama (treated)	$Y_{T,Pre}$	$Y_{T,Post}$
Arkansas (control)	$Y_{C,Pre}$	$Y_{C,Post}$

- Goal: estimate the effect of lowering the drinking age

The 2×2 calculation

- Two obvious estimators, neither of them good:
  - Cross-section in Post: $Y_{T,Post} - Y_{C,Post}$
    Confounded by fixed differences between Alabama and Arkansas
  - Time-series in Alabama: $Y_{T,Post} - Y_{T,Pre}$
    Confounded by other things changing over time

- DD combines them:

$$\delta_{DD} = \left(Y_{T,Post} - Y_{T,Pre}\right) - \left(Y_{C,Post} - Y_{C,Pre}\right)$$

- Differences out fixed unit differences, and differences out common time shocks

Potential outcomes notation

- For each unit at each time, two potential outcomes:
  - $Y_{it}(1)$ if the unit is treated
  - $Y_{it}(0)$ if the unit is not treated
- We observe one of them, not both:

$$Y_{it} = D_{it} \cdot Y_{it}(1) + (1 - D_{it}) \cdot Y_{it}(0)$$

- Want to estimate the ATT (average treatment effect on the treated):

$$\text{ATT} = E\big[\, Y_{it}(1) - Y_{it}(0) \mid D_{it} = 1 \big]$$

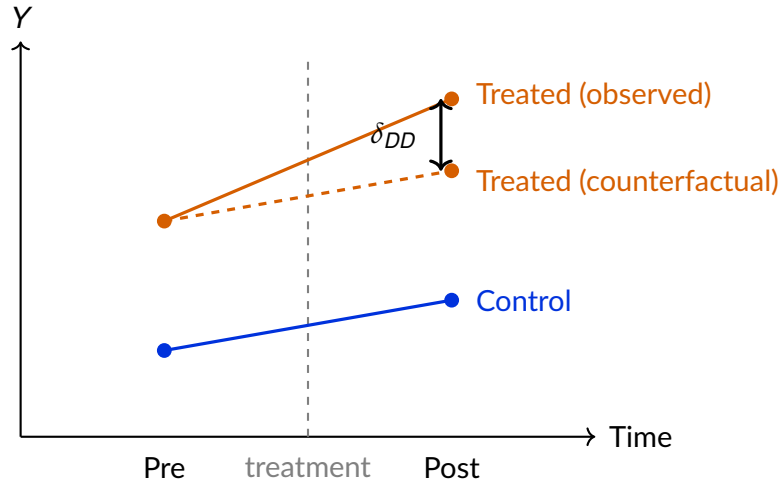
- Problem: $Y_{it}(0)$ for treated units in Post is the missing counterfactual
- Need an assumption about how it would have evolved

The diff-in-diff assumption

$$
\begin{aligned}
\delta_{DD} &= \underbrace{\left(Y_{T,Post} - Y_{T,Pre}\right)}_{\text{Pre-to-post change in T}} - \underbrace{\left(Y_{C,Post} - Y_{C,Pre}\right)}_{\text{Pre-to-post change in C}} \\
&= \left(Y_{T,Post}(1) - Y_{T,Pre}(0)\right) - \left(Y_{C,Post}(0) - Y_{C,Pre}(0)\right) \\
&= \underbrace{Y_{T,Post}(1) - Y_{T,Post}(0)}_{\text{ATT}} \\
&\quad + \underbrace{Y_{T,Post}(0) - Y_{T,Pre}(0) - \left(Y_{C,Post}(0) - Y_{C,Pre}(0)\right)}_{\text{= 0 under parallel trends}}
\end{aligned}
$$

Parallel trends: if treatment hadn't happened, outcomes in T and C would have moved by the same amount between Pre and Post.

Parallel trends, visualised



Parallel trends $\Leftrightarrow$ the dashed line is the right counterfactual.

The 2×2 as a regression

- With one treated and one control unit and two periods, run:

$$Y_{st} = \alpha + \beta \cdot \mathbb{1}[\text{Treat}_s] + \gamma \cdot \mathbb{1}[\text{Post}_t] + \delta \cdot \mathbb{1}[\text{Treat}_s] \times \mathbb{1}[\text{Post}_t] + e_{st}$$

- Take expectations in each cell:

$$\begin{aligned} E[Y \mid C, Pre] &= \alpha \\ E[Y \mid T, Pre] &= \alpha + \beta \\ E[Y \mid C, Post] &= \alpha + \gamma \\ E[Y \mid T, Post] &= \alpha + \beta + \gamma + \delta \end{aligned}$$

- $\hat{\delta}$ on the interaction equals the DD calculation

- Bonus: regression gives standard errors for free (more on those later)

We need more states, but...

- Let's add one more state: Tennessee
- Tennessee is messier:
  - Dropped its MLDA to 18 in 1971
  - Raised it to 19 in 1979
- Adding Tennessee breaks the 2×2 setup:
  - More than one treated state, treated at different times
  - No single "Post" period anymore
  - No single binary treatment status
- Need a regression that handles all this

Step 1: replace the interaction with a policy indicator

- With Alabama vs. Arkansas alone, $Treat_s \times Post_t = \mathbb{1}[\text{MLDA} < 21]_{st}$
- Replace the interaction with the policy indicator directly:

$$Y_{st} = \alpha + \beta \cdot \mathbb{1}[\text{Treat}_s] + \gamma \cdot \mathbb{1}[\text{Post}_t] + \delta \cdot \mathbb{1}[\text{MLDA} < 21]_{st} + e_{st}$$

- Same coefficient as before
- But now generalisable: the policy indicator can switch on at different times for different states, and switch off again

Step 2: many time periods, many treated units

- Replace $Post_t$ with a full set of year dummies
- Replace $Treat_s$ with a full set of state dummies

$$Y_{st} = \alpha + \sum_{\ell \neq 1} \beta_\ell \mathbb{1}[\text{state} = \ell] + \sum_{y \neq 1970} \gamma_y \mathbb{1}[\text{year} = y] + \delta \cdot \mathbb{1}[\text{MLDA} < 21]_{st} + e_{st}$$

- Cleaner with fixed-effects notation:

$$\boxed{Y_{st} = \alpha_s + \gamma_t + \delta \cdot \mathbb{1}[\text{MLDA} < 21]_{st} + e_{st}}$$

- This is the two-way fixed effects (TWFE) regression
- α_s absorbs anything constant about state s
- γ_t absorbs anything common to all states in year t
- δ identified from within-state changes in the policy

What TWFE does, intuitively

- There are no fixed treated/control states anymore
  - Each state is its own control during years it doesn't change policy
  - Each state is treated during years immediately after it changes policy
- Ditto for years
- Pools many small "natural experiments" into one estimate
- Identifying assumption is still parallel trends:
  - Absent the policy change, treated and untreated state-years would have moved similarly

Stata for TWFE

```
* Set up the panel
xtset state year

* TWFE with reghdfe (preferred)
reghdfe mrate under21, absorb(state year) cluster(state)

* Same thing with areg (slower for many FE)
areg mrate under21 i.year, absorb(state) cluster(state)

* Same thing with raw dummies (don't do this with millions of units)
reg mrate under21 i.state i.year, cluster(state)
```

- All three give the same point estimate
- Always cluster (next section)

Why cluster?

- Default OLS standard errors assume residuals are independent across observations
- In panels, this is almost never true:
  - Mortality rates in Alabama in 1975 and 1976 are correlated
  - Anything making 1975 a bad year persists into 1976
- If you ignore this, your standard errors are too small and t-stats are too big
- Particularly bad with policy variables that vary at the state level: treatment status doesn't change much within a state, so your effective N is much smaller than 51×14

Cluster at the unit level

- Bertrand, Duflo, Mullainathan (QJE 2004): clustering at the state level fixes the problem
  - Allows arbitrary serial correlation within state
  - Assumes independence across states
- Rule of thumb: need at least ≈ 30 clusters for the asymptotics to work
  - With 50 US states you're fine
  - With 10 Canadian provinces you are not. Use wild cluster bootstrap (`boottest`)
- Cluster on the unit even when the unit FE isn't in the regression
- In Stata: `vce(cluster state)` or `cluster(state)`

Common mistakes

- Clustering at the wrong level
  - Treatment varies at the state level $\Rightarrow$ cluster at state, not state-year
- Clustering with too few clusters and trusting the SEs
  - Wild bootstrap (`boottest`) is your friend
- Reporting only robust (non-clustered) SEs in a panel setting
  - Almost always wrong, and a referee will catch it

Recap

- Fixed effects let us absorb anything fixed about a unit, weakening the OLS exogeneity assumption
- Diff-in-diff compares pre-to-post changes in a treated group to pre-to-post changes in a control group
  - Identification rests on parallel trends
- Two-way fixed effects extends this to many units and many periods, pooling lots of small natural experiments
- Always cluster your standard errors at the unit level

Tomorrow

- Event study designs
  - Dynamic version of DD: how does the effect evolve over time?
  - How to test the parallel trends assumption (sort of)
  - The recent literature on staggered DD and what to do about it

- Then no class Monday (Victoria Day), Tables and Figures next Tuesday